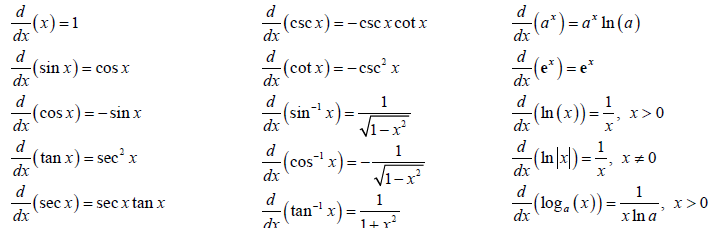
**Lesson 0**

Revision: Derivatives P.P.I.C.Q

*From MAT1613*

Derivatives are the slope of a function. It calculates the instantaneous rate of change at each point of the old function.

Common derivatives:



What is and what is the difference between and ?

is a function that takes one input.

Differentiation incomplete

*differentiation-with-respect-to-x*

or for brevity , is a function with its input y.

Differentiation complete

*the result of taking the derivative-with-respect-to-x of y*

**Lesson 0**

Differential Equation

<https://tutorial.math.lamar.edu/Classes/DE/Definitions.aspx>

A differential equation is any equation which contains derivatives, either ordinary derivatives or partial derivatives.

Example: Newton’s second law of motion

– force

– mass

– acceleration

Rewrite

or

Newton’s Second Law can now be written as a differential equation in terms of either the velocity, , or the position, , of the object as follows:

**Order**

The order of a differential equation is the largest derivative present in the differential equation.

**Ordinary Differential Equations** **(ODE)**

has ordinary derivatives in it

**Partial Differential Equations** **(PDE)**

has partial derivatives in it

Example:

Show that is a solution to for

*first derivative*

*second derivative*

*Substitute into ODE*

*Hence, satisfies the differential equation and is a solution*

**Initial conditions**

Initial Condition(s) are a condition, or set of conditions, on the solution that will allow us to determine which solution that we are after. Initial conditions are values of the solution and/or its derivative(s) at specific points. For the above example, there are actually an infinite number of solutions:

Therefore, the same example with initial conditions:

Example:

Show that is a solution to

, ,

is the only solution to this differential equation.

**Initial Value Problem**

An Initial Value Problem (or IVP) is a differential equation along with an appropriate number of initial conditions.

The following are examples of IVP’s:   
 , ,

,

**Interval of Validity**

The interval of validity for an IVP with initial condition(s)

//TODO

**General Solution**

The general solution to a differential equation is the most general form that the solution can take and doesn’t take any initial conditions into account.

Example: the general solution to is

**Implicit/Explicit Solution**

**Lesson 0**

Direction/Slope Fields

It’s impossible to find explicit formulas for solutions of some differential equations. Even if there are such formulas, they may be so complicated that they’re useless. In this case we may resort to graphical or numerical methods to get some idea of how the solutions of the given equation behave.

Slope

<https://www.youtube.com/watch?v=Wr9VOum9Co0>

|  |  |
| --- | --- |
| Gradient | Drawing |
|  |  |
| undefined |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

Example: draw the slope field of

*Remember that y-prime is dependent on*

Step 1: draw a table to show function values/gradients

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  | Explanation |
| -2 |  | -2 | *anywhere , the gradient is* |
| -1 |  | -1 | *anywhere , the gradient is* |
| 0 |  | 0 | *anywhere , the gradient is* |
| 1 |  | 1 | *anywhere , the gradient is* |
| 2 |  | 2 | *anywhere , the gradient is* |

Step 2: solve the differential equation

This will guide us on the shape of one of the solution curves

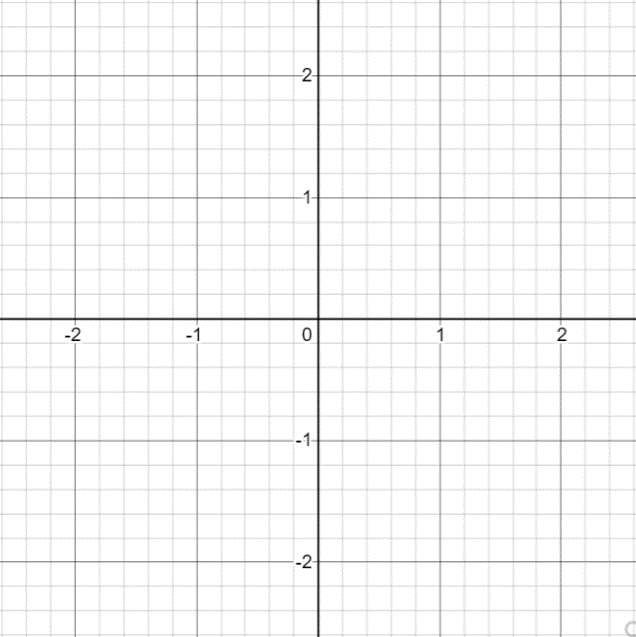
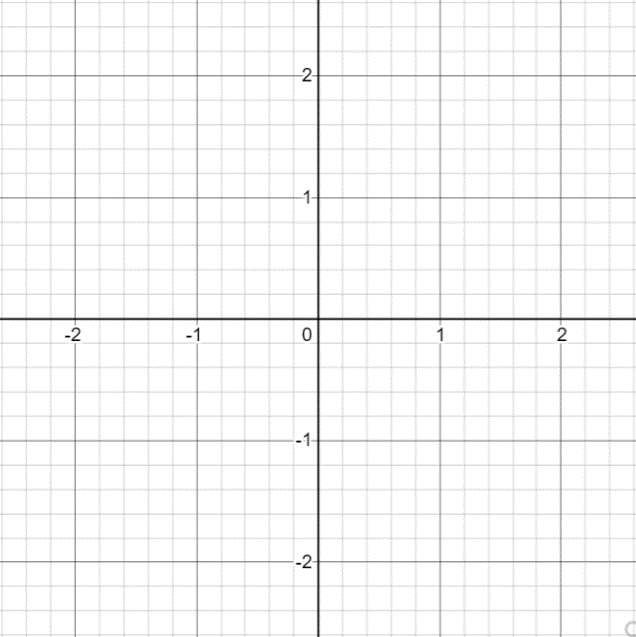
*is also*

*multiply both sides by*

*find the anti-derivative of both sides*

Step 3: draw the slope field

Then, draw one of the solution curves



Example: draw the slope field of

*Remember that y-prime is dependent on*

Step 1: draw a table to show function values/gradients

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  | Explanation |
|  | -2 | -2 | *anywhere y, the gradient is* |
|  | -1 | -1 | *anywhere y, the gradient is* |
|  | 0 | 0 | *anywhere y, the gradient is* |
|  | 1 | 1 | *anywhere y, the gradient is* |
|  | 2 | 2 | *anywhere y, the gradient is* |

Step 2: solve the differential equation

This will guide us on the shape of one of the solution curves

*is also*

*multiply both sides by*

*divide both sides by*

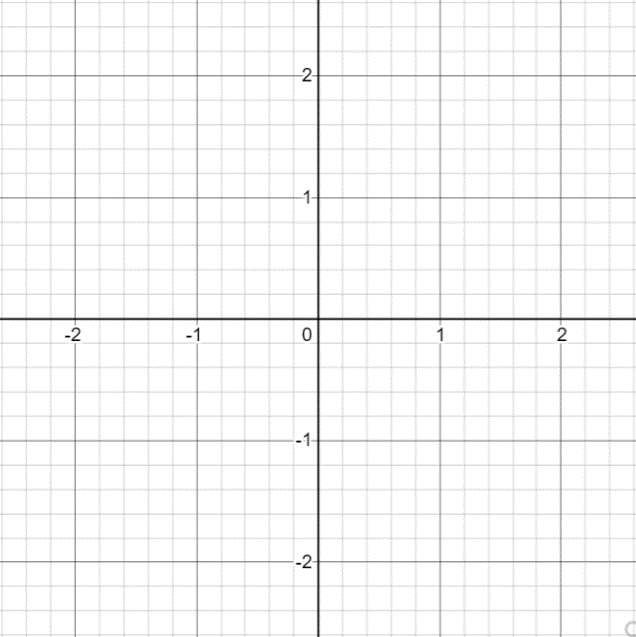
*divide both sides by*

*find sthe anti-derivative of both sides*

*is also just a constant c*

Step 3: draw the slope field

Then, draw one of the solution curves



Example: draw the slope field of

Step 1: draw a table to show function values/gradients

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  | Explanation |
| 0 | 0 | 0 | *anywhere y, the gradient is* |
| 1 | 1 | 0 | *anywhere y, the gradient is* |
| 2 | 2 | 0 | *anywhere y, the gradient is* |
|  |  |  |  |
| 1 | 0 | 1 | *Pattern for solution curve* |
| 2 | 1 | 1 | *Pattern for solution curve* |
| 0 | -1 | 1 | *Pattern for solution curve* |
|  |  |  |  |
| 2 | 0 | 2 | *Pattern under solution curve* |
| 3 | 1 | 2 | *Pattern under solution curve* |
|  |  |  |  |
| 0 | 1 | -1 | *Pattern above solution curve* |
| 1 | 2 | -1 | *Pattern above solution curve* |
|  |  |  |  |
| 0 | 2 | -2 |  |
| 3 | 0 | 3 |  |

*Wherever , y-prime is 0*

Step 2: solve the differential equation

This will guide us on the shape of one of the solution curves

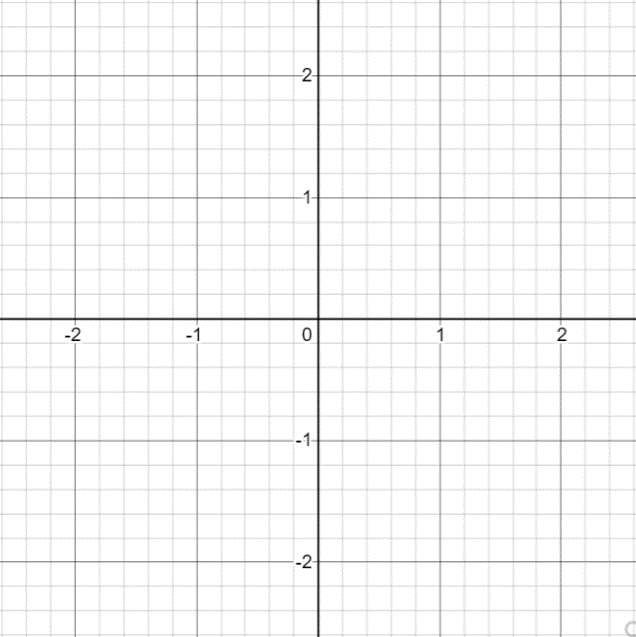
*is also*

**ODE: first order linear**

**//TODO**

Step 3: draw the slope field

Then, draw one of the solution curves



**Lesson 0**

Ordinary Differential Equations (ODE) - S.H.I.E

<https://www.youtube.com/watch?v=IFpT-Ptmkyg&ab_channel=MathbyLEO>

has ordinary derivatives in it

Different methods to solve ODE’s:

[1] 1 separate x terms and y terms

[2] integrate both sides

Implicit: (y-terms).dy=(x-terms).dx + c

Explicit: y = mx+c

[1] Seperable Equations

*Write in standard form*

[2] Homogenous method

[1] check if homogenous

f(kx,ky) = f(x,y)

*Identify the functions and*

[1] find P(x)

[2] find M(x) (aka integrating factor I(x))

[3] multiply equation by M(x)

[3] Integrating factor

*Identify the Integrating factor*

or

[4] Exact Differential Equations

[1] identify if it is standard form

M(x,y).dx + N(x,y).dy = 0

*Or*

Remember:

[2] identify M an N

[3] Check if it is exact DE

[4] Integrate M or N

*Write the general solution*

**ODE: first order linear**

example: Solve for

*write in standard form*

**ODE: first order linear**

example: Solve for

*write in standard form*